

JEE MAIN 2026

Sample Paper – 1

Time Allowed: 3 hours

Maximum Marks: 300

General Instructions:

1. The test consists of total **75 questions**.
2. Each subject (**PCM**) has **25 questions**.
3. Each subject divided into two sections. **Section A** consists of 20 multiple-choice questions & **Section B** consists of 5 numerical value-type questions.
4. **Marking Scheme:**
 - **Section A (MCQs):** +4 marks for each correct answer, –1 mark for each incorrect answer, 0 marks for unattempted.
 - **Section B (Numerical):** +4 marks for each correct answer, 0 marks for incorrect or unattempted.
5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.



Physics

PART – A

SECTION – A (One Options Correct Type)

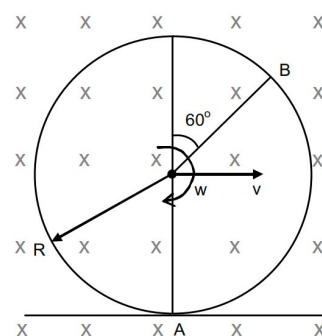
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. In order to convert a milliammeter of range 1.0 mA and resistance 1.0 ohm into a voltmeter of range 10 V, a resistance of how many ohms should be connected with it and in what manner?

(A) 999 Ω in series (B) 999 Ω in parallel
(C) 9,999 Ω in series (D) 9,999 Ω in parallel

2. A conducting wheel is rolling on the ground in a uniform magnetic field B_0 then the emf induced between points A and B; $V_A - V_B$ will be

(A) $B_0 w^2 (\sqrt{3} R)$
(B) $-\frac{\sqrt{3}}{2} B w^2 R$
(C) $B_0 w^2 \sqrt{2} R$
(D) $-B_0 w^2 \left(\frac{\sqrt{5}}{2} \right) R$

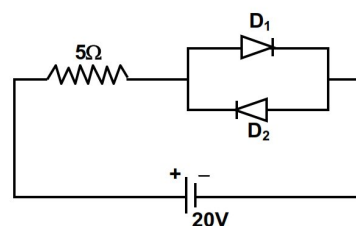


3. Forces acting on a particle moving in a straight line varies with the velocity of the particle as $F = \frac{\alpha}{v}$ where α is constant. The work done by this force in time interval Δt is

(A) $\alpha \Delta t$ (B) $\frac{1}{2} \alpha \Delta t$
(C) $2 \alpha \Delta t$ (D) $\alpha^2 \Delta t$

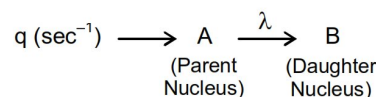
4. In the figure shown below, chose the correct answer.

(A) Voltage across D_2 is 20 Volt.
(B) Voltage across D_1 is 20 Volt.
(C) Current through diode D_1 is 4A.
(D) Current through diode D_1 is 0 A.



5. In a radioactive reaction an unstable nucleus A dis-integrates into a stable nucleus B. But A is generated at a constant rate of q nucleus per second. Then at steady state number of nucleus of A will be

(A) $q\lambda$ (B) $\frac{q}{\lambda}$
(C) $q - \lambda$ (D) $\frac{\lambda}{q}$

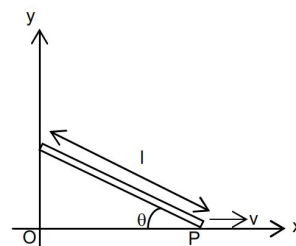


6. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monatomic gas of molar mass M_A . Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B . Both gases are at the same temperature. If the frequency of the second harmonic in pipe A is equal to the frequency of the third harmonic in pipe B, what is the value of M_A/M_B ?

- (A) $\frac{100}{189}$ (B) $\frac{200}{189}$
(C) $\frac{400}{189}$ (D) $\frac{500}{189}$

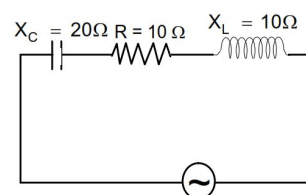
7. The end of a rigid rod of length l is moved with a constant velocity v . Find the speed of the midpoint of the rod

- (A) $\frac{v}{2 \sin \theta}$ (B) $\frac{v}{2} \tan \theta$
(C) $\frac{v}{2} \cot \theta$ (D) $\frac{v}{2 \cos \theta}$



8. In LCR circuit shown in figure, then choose incorrect option

- (A) Current will lead the voltage
(B) rms value of current is 20 A
(C) Power factor of the circuit is $\frac{1}{\sqrt{2}}$
(D) Voltage drop across resistance = voltage drop across inductor

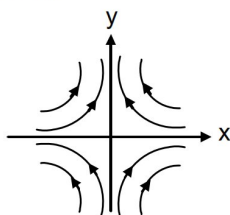


9. A tunnel is dug along diameter inside the surface of earth and a particle is projected from the centre of tunnel. The minimum velocity of particle such that it escape out from the earth's gravitation field is (Radius of earth = R_e)

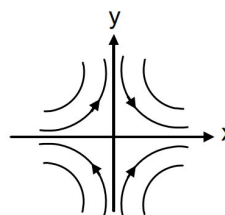
- (A) $\sqrt{2gR_e}$ (B) $\sqrt{\frac{3}{2}gR_e}$
(C) $\sqrt{3gR_e}$ (D) $\sqrt{\frac{5}{2}gR_e}$

10. Electric potential is varying with x and y as $V = 2(x^2 - y^2)$. The corresponding field pattern is:

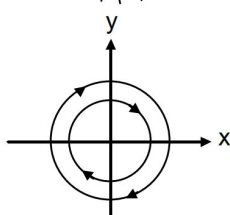
(A)



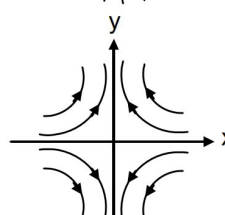
(B)



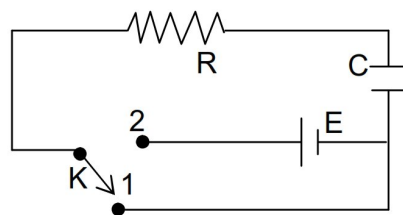
(C)



(D)



11. In the shown circuit involving a resistor of resistance $R \Omega$, capacitor of capacitance C farad and an ideal cell of emf E volt, the capacitor is initially uncharged and the key is in position 1. At $t = 0$ second the key is pushed to position 2 for $t_0 = RC$ seconds and then key is pushed back to position 1 for $t_0 = RC$ seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.



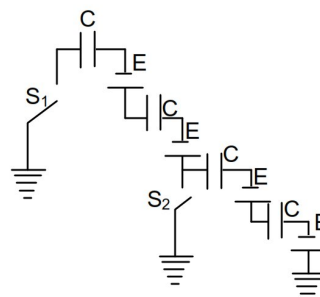
The charge on capacitor at $t = 2RC$ second is

- (A) CE (B) $CE \left(1 - \frac{1}{e}\right)$
 (C) $CE \left(\frac{1}{e} - \frac{1}{e^2}\right)$ (D) $CE \left(1 - \frac{1}{e} + \frac{1}{e^2}\right)$
12. Two polaroids are placed in the path of unpolarized beam of intensity I_0 such that no light is emitted from the second polaroid. If a third Polaroid whose polarization axis makes an angle θ with the polarization axis of first Polaroid, is placed between these polaroids then the intensity of light emerging from the last polaroid will be

- (A) $\left(\frac{I_0}{8}\right) \sin^2 2\theta$ (B) $\left(\frac{I_0}{4}\right) \sin^2 2\theta$
 (C) $\left(\frac{I_0}{2}\right) \cos^4 \theta$ (D) $I_0 \cos^4 \theta$

13. In the given circuit, all the capacitors are initially uncharged. After closing the switch S_1 for a long time suddenly S_2 is also closed and kept closed for a long time. Total heat produced after closing S_2 will be :

- (A) $4C\varepsilon^2$ (B) $\frac{1}{2}C\varepsilon^2$
 (C) $2C\varepsilon^2$ (D) 0



14. The resistivity of pure silicon is $2300 \Omega\text{-m}$ and the mobilities of electrons and holes in it are 0.135 and $0.048 \text{ m}^2/\text{V-s}$ respectively. The resistivity of a specimen of silicon doped with 10^{19} atoms of phosphorus per meter will be

- (A) $4.6 \Omega \text{ m}$ (B) $4 \Omega \text{ m}$
 (C) $4.4 \Omega \text{ m}$ (D) $0 \Omega \text{ m}$

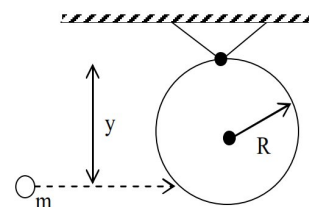
15. A perfectly absorbing surface intercepts a parallel beam of monochromatic light of power 10 W ($\lambda = 500 \text{ nm}$) incident on it normally the force exerted by light beam on the surface is

- (A) $\frac{1}{4} \times 10^{-7} \text{ N}$ (B) $\frac{1}{3} \times 10^{-7} \text{ N}$
 (C) $\frac{1}{2} \times 10^{-7} \text{ N}$ (D) $1 \times 10^{-7} \text{ N}$

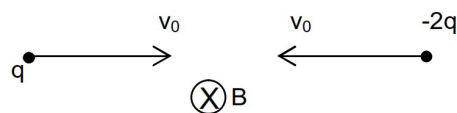
16. The efficiency of a Carnot cycle is $1/6$. If on reducing the temperature of the sink by 65°C , the efficiency becomes $1/3$, the initial temperatures between which the cycle is working are
 (A) 390 K, 325 K (B) 780 K, 325 K
 (C) 390 K, 162 K (D) 300 K, 100 K

17. A spherical object of mass 1 kg and radius 1 m is falling vertically downward inside a viscous liquid in a gravity free space. At a certain instant the velocity of the sphere is 2 m/s. If the coefficient of viscosity of the liquid is $\frac{1}{18\pi}$ N-S/m², then velocity of ball will become 0.5 m/s after a time
 (A) $\ln 4$ s (B) $2 \ln 4$ s
 (C) $3 \ln 4$ s (D) $2 \ln 2$ s

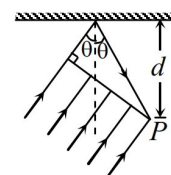
18. A small ball of mass m hits the cylinder which is hinged at top and free to rotate in vertical plane as shown in figure. Mass of cylinder is M . Linear momentum of system (ball + cylinder)
 (A) Remains conserved if $y = R$
 (B) Remains conserved if $y = \frac{3}{2}R$
 (C) Remains conserved if $y = \frac{7}{5}R$
 (D) Cannot be conserved for any value of y .



19. A charge particle q of mass m is placed at a distance d from another charge particle $-2q$ of mass $2m$ in a uniform magnetic field B as shown. If particles are projected towards each other with equal speed v_0 , so that the two particles touches each other without collision during its motion. (Assume only force due to magnetic field acts on the particle)



- (A) $\frac{qBd}{m}$ (B) $\frac{qBd}{2m}$
 (C) $\frac{2qBd}{m}$ (D) $\frac{3qBd}{2m}$
20. A plane wave front of light is incident on a plane mirror. Intensity is maximum at P when
 (A) $\cos \theta = \frac{\lambda}{2d}$ (B) $\cos \theta = \frac{3\lambda}{4d}$
 (C) $\sec \theta - \cos \theta = \frac{3\lambda}{4d}$ (D) $\sec \theta - \cos \theta = \frac{\lambda}{2d}$

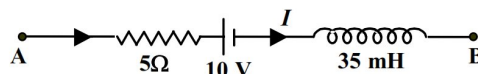


SECTION – B

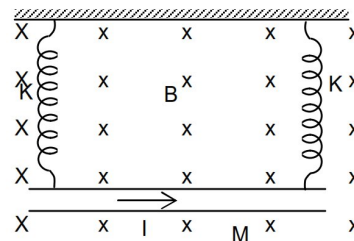
(Numerical Answer Type)

This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. The network shown in Figure is part of a complete circuit. If at a certain instant the current (I) is 5A, and is decreasing at a rate of 10^3 A/s, then $V_B - V_A =$



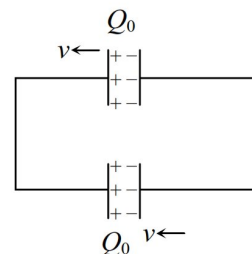
22. A metal rod of mass 10gm and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 ampere current passes through the rod it rises by 1 cm. The magnetic field is $x \times 10^{-2}$ T ($g = 10$ m/s²). Find the value of $10x$.



23. A concave mirror of focal length 15 cm is placed in air at a height of 45 cm above the water surface as shown. The axis of the mirror is vertical and the reflecting surface faces the water surface. A point object is placed on the axis of concave mirror at a distance 35 cm above the water surface. Find the position of image formed after two successive processes- first reflection from the concave mirror and then first refraction from the water (refractive Index of water 4/3). The distance of image from water surface (in cm) is



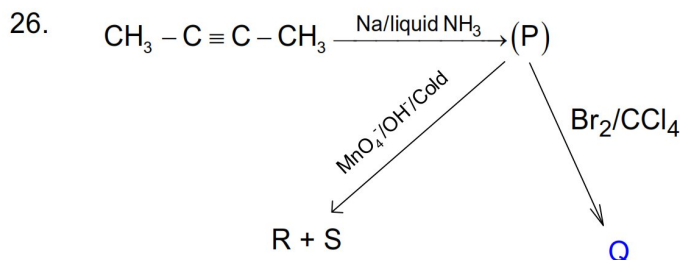
24. Two identical capacitor connected as shown and having initial charge Q_0 . Separation between plates of capacitor is d_0 . Suddenly the left plate of upper capacitor and right plate of lower capacitor start moving with speed v towards left while other plate of capacitor remains fixed. (given $\frac{Q_0 v}{2d} = 1$ amp). Find the value of current (in amp) in the circuit.



25. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms⁻¹. Find the kinetic energy (in mJ) of a particle of mass 6 kg moving with same velocity as that of centre of mass of the above two particles of mass 1 kg and 5 kg.

SECTION – A
(One Options Correct Type)

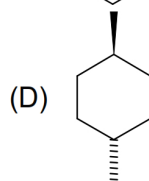
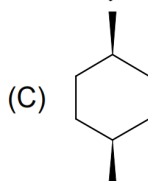
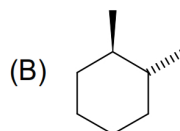
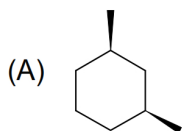
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.



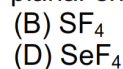
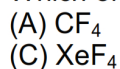
Which of the following statement is correct about the products of above reaction?

- (A) 'Q' contains no asymmetric carbon atom
 (B) 'P' and 'Q' are functional isomers
 (C) 50% of R and 50% of Q makes a racemic mixture
 (D) 'P' has one stereoisomer
27. What is the wavelength of the motion of the electron of hydrogen atom along the fourth orbit?
 [r_0 = Radius of first orbit of 'H' atom]
 (A) $2\pi r_0$ (B) $4\pi r_0$
 (C) $8\pi r_0$ (D) $16\pi r_0$
28. $\text{SrCO}_3 (\text{s}) \rightleftharpoons \text{Sr}^{2+} (\text{aq}) + \text{CO}_3^{2-} (\text{aq})$
 Addition of which of the following substance can increase the solubility of SrCO_3 according to above reaction?
 (A) NaCl (B) HCl
 (C) K_2CO_3 (D) NaOH
29. Which of the following is not identical between inorganic benzene and organic benzene?
 (A) Number of atoms (B) Number of electrons
 (C) Number of sigma bonds (D) Number of polar covalent bonds
30. Which of the following is the strongest acid?
 (A) $\text{CH}_3\underset{\text{Cl}}{\text{CH}}\underset{\text{F}}{\text{CH}_2}\text{CHCOOH}$ (B) $\text{CH}_3\underset{\text{F}}{\text{CH}}\underset{\text{Cl}}{\text{CH}_2}\text{CHCOOH}$
 (C) $\text{CH}_3\text{CH}_2\underset{\text{F}}{\text{CH}}\underset{\text{Cl}}{\text{CHCOOH}}$ (D) $\text{CH}_3\text{CH}_2\underset{\text{Cl}}{\text{CH}}\underset{\text{F}}{\text{CHCOOH}}$
31. Which of the following are heating products of borax?
 (A) Na_3B and NaBO_2 (B) Na_2O and B_2H_6
 (C) NaBO_2 and B_2O_3 (D) Na_3B and B_2O_3

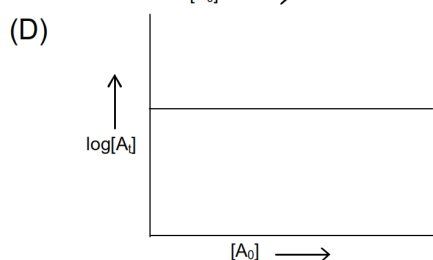
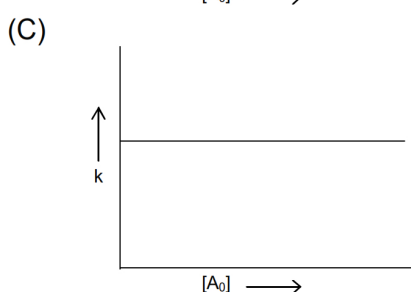
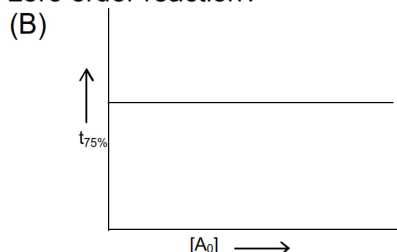
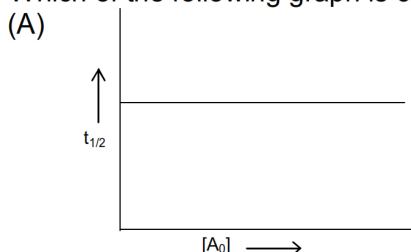
32. Which of the following compounds has maximum number of stereoisomers?



33. Which of the following molecule has square planar shape?

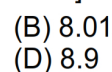
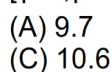


34. Which of the following graph is correct for a zero order reaction?

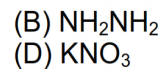
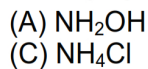


35. What is the pH of 10^{-2} M NaH_2PO_4 ?

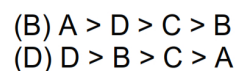
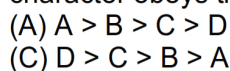
[$p^{K_{a1}}$, $p^{K_{a2}}$ and $p^{K_{a3}}$ of H_3PO_4 are 7.2, 10.6 and 12.8]



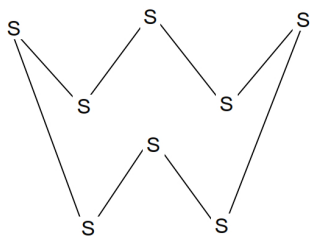
36. In which of the following compound, nitrogen exhibits its lowest negative oxidation state?



37. Heat of neutralization of four monobasic acids A, B, C and D are -13.7, -9.4, -11.2 and -12.4 Kcal / mole respectively, when they are neutralized by a common base. The acidic character obeys the order



38. The oxidation number and hybridization of sulphur in S_8 molecule are respectively:

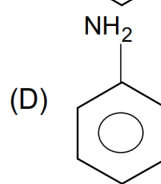
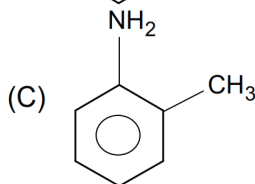
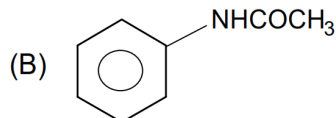
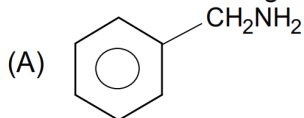


- (A) +2 and sp^2 (B) +2 and sp^3
(C) zero and sp^2 (D) zero and sp^3

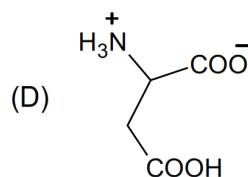
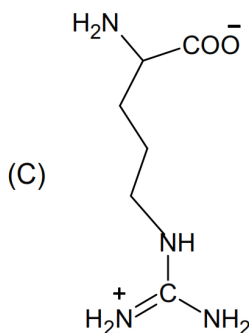
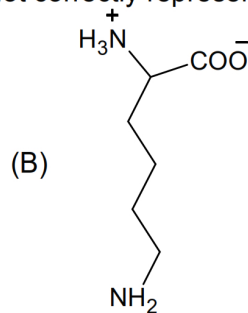
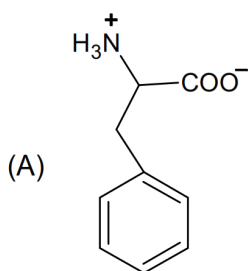
39. The relation between the molar conductance ($\wedge_m$) and equivalent conductance ($\wedge_e$) of the solution of $MgCl_2 \cdot KCl \cdot 6H_2O$ is

- (A) $\wedge_m = \wedge_e \times 8$ (B) $\wedge_m = \wedge_e \times 3$
(C) $\wedge_e = \wedge_m \times 3$ (D) $\wedge_e = \wedge_m \times 8$

40. Which of the following is the strongest base?



41. Which of the following zwitter ionic form is not correctly represented for an amino acid?



42. Which one of the following has aromatic character ?
 (A) Cyclopentadienyl cation (B) Cyclopentadienyl radical
 (C) Cyclopentadienyl anion (D) Cyclopentadiene
43. Which of the following interhalogen compound does NOT exist?
 (A) IF₃ (B) IF₄
 (C) IF₅ (D) IF₇
44. The fraction of chlorine precipitated by AgNO₃ solution from [Cr(NH₃)₅Cl]Cl₂ is
 (A) $\frac{1}{2}$ (B) $\frac{1}{3}$
 (C) $\frac{2}{3}$ (D) $\frac{1}{4}$
45. The following two equilibria exist simultaneously in a closed container of volume 2 L
 $A_2(g) + B_2(g) \rightleftharpoons 2AB(g)$
 $A_2(g) + C(g) \rightleftharpoons A_2C(g)$
 Which of the following change does not affect the concentration of AB(g)
 (A) Increase in pressure (B) Addition of C
 (C) Addition of He gas at constant volume (D) All of these

SECTION – B

(Numerical Answer Type)

This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

46. pH of 10⁻³ M solution of acetic acid ($K_a = 1 \times 10^{-5}$) is
47. For the reaction at 25°C
 $A(g) + 2B(g) \rightleftharpoons 2C(g) + D(g)$
 The initial concentration of B is 1.5 times the molar concentration of A. If at equilibrium, the concentration of A and D are equal. The equilibrium constant (K_c) at 25°C is
48. How many maximum number of electrons can be accommodated in the 9th period of the periodic table?
49. $A(g) \longrightarrow B(g) + 3C(g) + 4D(g)$
 Above first order reaction takes place by taking A(g) in a container at 100 mm of Hg. What will be the total pressure in the container after one half-life period in mm of Hg unit?
50. $FeCl_3(aq) + K_4[Fe(CN)_6](aq) \longrightarrow (P)$
 (Blue colour aqueous solution)
 (P) is an ionizable complex salt. Reaction of the cation of (P) with NaOH forms a reddish brown precipitate. How many moles of NaOH required for complete reaction of all the cations obtained from (P)?

SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

51. Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100} \right] n$, where $[n]$ denotes the greatest integer less than or equal to n .

Then $\sum_{n=1}^{56} f(n)$ is equal to:

- (A) 56 (B) 689
(C) 1287 (D) 1399

52. If α and β are the roots of the quadratic equation, $x^2 + x \sin \theta - 2 \sin \theta = 0$, $\theta \in \left(0, \frac{\pi}{2} \right)$ then

$\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$ is equal to:

- (A) $\frac{2^{12}}{(\sin \theta + 8)^{12}}$ (B) $\frac{2^{12}}{(\sin \theta - 4)^{12}}$
(C) $\frac{2^{12}}{(\sin \theta - 8)^6}$ (D) $\frac{2^6}{(\sin \theta + 8)^{12}}$

53. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$ is equal to:

- (A) $\frac{1}{(1 + \cot^3 x)} + c$ (B) $-\frac{1}{3(1 + \tan^3 x)} + c$
(C) $\frac{\sin^3 x}{(1 + \cos^3 x)} + c$ (D) $-\frac{\cos^3 x}{3(1 + \sin^3 x)} + c$

54. If the tangent at a point P, with parameter t , on the curve $x = 4t^2 + 3, y = 8t^3 - 1, t \in \mathbb{R}$, meets the curve again at a point Q, then the coordinates of Q are:

- (A) $(16t^2 + 3, -64t^3 - 1)$ (B) $(4t^2 + 3, -8t^3 - 2)$
(C) $(t^2 + 3, t^3 - 1)$ (D) $(t^2 + 3, -t^3 - 1)$

55. Let the system of linear equations
 $x + y + kz = 2$
 $2x + 3y - z = 1$
 $3x + 4y + 2z = k$
 have infinitely many solutions. Then the system
 $(k + 1)x + (2k - 1)y = 7$
 $(2k + 1)x + (k + 5)y = 10$ has :
 (A) infinitely many solutions (B) unique solution satisfying $x - y = 1$
 (C) no solution (D) unique solution satisfying $x + y = 1$
56. Let S be the set of all solutions of the equation
 $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ is equal to
 (A) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (B) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$
 (C) $\frac{-2\pi}{3}$ (D) 0
57. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2 , then $8\sigma^2$ is equal to
 (A) 220 (B) 210
 (C) 200 (D) 105
58. Let $\beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$ for some $\alpha \in \mathbb{R}$. Then the value of $\alpha + \beta$ is:
 (A) $\frac{14}{5}$ (B) $\frac{3}{2}$
 (C) $\frac{5}{2}$ (D) $\frac{7}{2}$
59. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ ($b < 5$) and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pairs (α, β) is equal to:
 (A) (8, 12) (B) $\left(\frac{20}{3}, 12\right)$
 (C) $\left(\frac{24}{5}, 10\right)$ (D) (8, 10)

60. The length of the minor axis (along y - axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is:
- (A) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (B) $\sqrt{\frac{5}{6}}$
 (C) $\frac{1}{2}\sqrt{\frac{5}{3}}$ (D) $\frac{1}{3}\sqrt{\frac{11}{3}}$
61. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?
- (A) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (B) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$
 (C) $(-3, -9)$ (D) $(-3, -8)$
62. In an isosceles triangle ABC, the vertex A is $(6, 1)$ and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid $\triangle ABC$, then $15(\alpha + \beta)$ is equal to:
- (A) 39 (B) 41
 (C) 51 (D) 63
63. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \dots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is:
- (A) 198 (B) 202
 (C) 212 (D) 218
64. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1-x)^8 + \dots + x^{10}$ is:
- (A) 120 (B) 420
 (C) 330 (D) 210
65. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S:
- (A) contains at least four elements (B) is a singleton
 (C) is an empty set (D) contains exactly two elements
66. Let nC_r denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10} (2^2 + 3k) {}^{10}C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$, $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ equal to _____.
- (A) 10 (B) 18
 (C) 19 (D) 20

67. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____
 (A) 50 (B) 28
 (C) 56 (D) 60
68. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then $2|\vec{c}|$ is equal to:
 (A) $\sqrt{55}$ (B) $\sqrt{37}$
 (C) $\sqrt{51}$ (D) $\sqrt{43}$
69. Let $\vec{a}$ and $\vec{b}$ be two vectors. Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is
 (A) -24 (B) -48
 (C) -84 (D) -60
70. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$, $\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$ such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then $\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c})$ is equal to _____.
 (A) 1 (B) 2
 (C) 3 (D) 4

SECTION – B

(Numerical Answer Type)

This section contains 05 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

71. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is _____
72. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \leq 2 \\ 0 & \text{if } |x| > 2 \end{cases}$
 Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in $\mathbb{R}$ where g is not continuous and not differentiable, respectively, then $n + m$ is equal to _____

73. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β , then $\beta - \alpha$ is equal to _____
74. Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y \text{ is odd positive integer or } x - y = 2\}$. The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to _____
75. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2 . Then $\alpha + r$ is equal to _____

SOLUTIONS

Physics

PART – A

SECTION – A

1. C
Sol. $10^{-3}(1 + R) = 10$
 $\therefore R = 9999\Omega$
2. B
Sol. $\Delta V = \frac{B\omega^2(\ell)}{2} = \frac{B\omega^2(\sqrt{3}R)}{2} = V_B - V_A$
3. A
Sol. $W = \int Fv dt$
4. C
Sol. Since, D_1 will be acting as short circuit, D_2 will act as open circuit so,
Current through 5Ω resistance and diode D_1 is $\frac{20}{5} = 4 \text{ A}$
5. B
Sol. At steady state.
Rate of generation of A = Rate of decay of A .
 $q = \lambda N_A$
 $\Rightarrow N_A = \frac{q}{\lambda}$



6. C

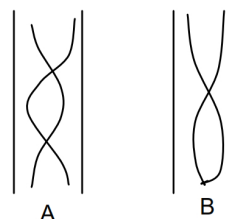
Sol. (a) $2 \frac{V_A}{2L_A} = 3 \frac{V_B}{4L_B}$

$$\Rightarrow \frac{V_A}{V_B} = \frac{3}{4} \quad [\because L_A = L_B]$$

$$V_A = \sqrt{\frac{\gamma_A RT}{M_A}}, \quad V_B = \sqrt{\frac{\gamma_B RT}{M_B}}$$

$$\Rightarrow \sqrt{\frac{\gamma_A}{\gamma_B} \frac{M_B}{M_A}} = \frac{3}{4}, \quad \gamma_A = \frac{5}{3}, \quad \gamma_B = \frac{7}{5}$$

$$\Rightarrow \frac{M_A}{M_B} = \frac{400}{189}$$



7. A

Sol. $\omega = \frac{v}{\ell \sin \theta}$

$$V_c = \frac{v}{2 \sin \theta}$$

8. B

Sol. $\tan \theta = \frac{X_c - X_L}{X_R}$

and $\cos \theta = \text{power factor}$.

9. C

Sol. Potential energy of particle at the centre of earth is $U = -\frac{3}{2} \frac{GMm}{R_e}$

$$\text{So, } V_e = \sqrt{\frac{3GM}{R_e}} = \sqrt{3gR_e}$$

10. D

Sol. $V = 2(x^2 - y^2)$

Equipotential surfaces will be hyperbolic and $\vec{E}$ everywhere will be $\perp$ to them.

11. C

Sol. For charging $q = CE(1 - e^{-t/RC})$

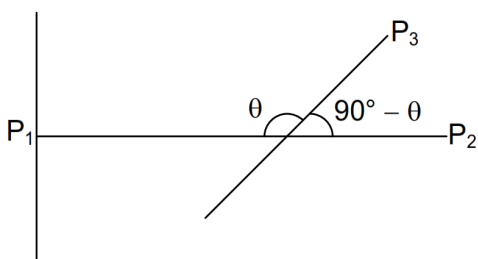
Charge at $t = RC \Rightarrow q_0 = CE(1 - e^{-1})$

At $t = RC$ discharging starts

$$\Rightarrow q = q_0(e^{-t/RC}) = CE(1 - e^{-1}) \times \frac{1}{e} = CE \left(\frac{1}{e} - \frac{1}{e^2} \right)$$

12. A

Sol. No light is emitted from the second polaroid, so P_1 and P_2 are perpendicular to each other



Let the initial intensity of light is I_0 . So Intensity of light after transmission from first polaroid $= \frac{I_0}{2}$.

Intensity of light emitted from P_3

$$I_1 = \frac{I_0}{2} \cos^2 \theta$$

Intensity of light transmitted from polaroid i.e., from

$$P_2 = I_1 \cos^2 (90^\circ - \theta) = \frac{I_0}{2} \cos^2 \theta \sin^2 \theta$$

$$P_2 = \frac{I_0}{8} (2 \sin \theta \cos \theta)^2 = \frac{I_0}{8} \sin^2 2\theta$$

13. D

Sol. No heat will be produced as no charge flows through S_2 when it is closed.

14. A

Sol. The resistivity of pure silicon is $2300 \Omega \cdot m$ and $\mu_e = 0.135 \text{ m}^2/\text{V}\cdot\text{s}$, $\mu_h = 0.048 \text{ m}^2/\text{V}\cdot\text{s}$.

Using

$$\sigma = 1/\rho = (n_i \mu_e + n_i \mu_h)e$$

$$(2300)^{-1} = n_i (0.135 + 0.048) \times 1.6 \times 10^{-19}$$

$$n_i = 1.5 \times 10^{16} / \text{m}^3.$$

Is the intrinsic electron & hole concentration. The resistivity of a specimen doped with 10^{19} P-atoms/ m^3 can be found from :

$$\sigma (\text{conductivity}) = n_e e \cdot \mu_e \quad (\because n_e = 10^{19} / \text{m}^3 \gg n_i)$$

$$= 10^{19} \times 1.6 \times 10^{-19} \times 0.135 = 0.216 \text{ mho} / \text{m}.$$

$$\rho = \frac{1}{\sigma} = 4.6 \Omega \cdot \text{m}$$

15. B

Sol. Power = 10W

$$\Rightarrow \text{Force} = \frac{P}{C} \quad (\text{for perfectly absorbing surface})$$

$$F = \frac{10}{3 \times 10^8 \text{ m/s}} = \frac{1}{3} \times 10^{-7}$$

16. A

Sol. Given $\eta_1 = \frac{1}{6}$, $\eta_2 = \frac{1}{3}$

If the temperatures of the source and the sink between which the cycle is working are T_1 and T_2 , then the efficiency in the first case will be

$$\eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{6}$$

In the second case $\eta_2 = 1 - \frac{T_2 - 65}{T_1} = \frac{1}{3}$

Solving $T_1 = 390 \text{ K}$ and $T_2 = 325 \text{ K}$.

17. C

Sol. $m \frac{dv}{dt} = -6\pi\eta r v$

$$(1) \frac{dv}{dt} = 6\pi \left(\frac{1}{18\pi} \right) (1) v$$

$$\frac{dv}{dt} = -\frac{v}{3}$$

$$\frac{dv}{v} = -\frac{dt}{3}$$

$$[\ln v]_2^{0.5} = \left[-\frac{t}{3} \right]_{t_1}^{t_2}$$

$$\ln 4 = \frac{\Delta t}{3}$$

$$\Delta t = 3 \ln 4$$

18. B

Sol. $mu_y = mVy + \frac{3}{2}MR^2\omega$ where u is speed of ball before collision, V speed of ball after collision and ω is angular speed of cylinder after collision.

$$mu = mV + MR\omega \text{ (COM)}$$

$$\therefore y = \frac{3R}{2}$$

19. B

Sol. $d = \frac{mv_0}{qB} + \frac{2mv_0}{2qB}$

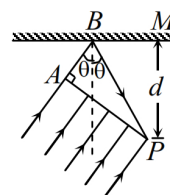
20. B

Sol. $BP = \frac{MP}{\cos \theta}$

$$AB = BP \cos 2\theta = \frac{MP}{\cos \theta} \cos 2\theta$$

$$AB + BP = n\lambda + \frac{\lambda}{2}$$

$$\frac{d \cos 2\theta}{\cos \theta} + \frac{d}{\cos \theta} = \frac{d}{\cos \theta} (2 \cos^2 \theta - 1 + 1) = 2d \cos \theta = n\lambda + \frac{\lambda}{2} \text{ for } n = 1, \cos \theta = \frac{3\lambda}{4d}$$



SECTION – B

21. 0

Sol. $V_A - V_B = i(5\Omega) + 10V + L \frac{di}{dt}$
 $= 5(5) + 10 + L(-10^{+3})$
 $V_A - V_B = 35 - 35 \times 10^{-3} \times 10^{+3} = 0.$

22. 15

Sol. Initially the rod will be in equilibrium if

$$2T_o = Mg \text{ with } T_o = kx_o \quad \dots(i)$$

when the current I is passed through the rod, it will experience a force

$F = BIL$ vertically up,

In equilibriums

$$2T + BIL = Mg \text{ with } T = kx \quad \dots(ii)$$

from (i) & (ii)

$$\frac{T}{T_o} = \frac{Mg - BIL}{Mg} \text{ i.e. } \frac{x}{x_o} = 1 - \frac{BIL}{Mg}$$

$$\text{or, } B = \frac{Mg(x_o - x)}{I L x_o}$$

Putting the values we get $B = 1.5 \times 10^{-2} T.$

23. 100

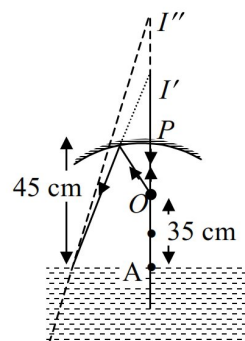
Sol. For the reflection at the concave mirror,

$$u = -10 \text{ cm}; v = ?; f = -15 \text{ cm}$$

From the mirror formula, we have

$$v = \frac{uf}{u-f} = \frac{(-10) \times (-15)}{-10+15} = \frac{150}{5} = +30 \text{ cm}$$

The positive sign indicates that the image is formed on the other side of the concave mirror,



Now, the image formed by the concave mirror serves as a virtual object for refraction at water surface which takes place from air to water. So,

$$\mu = \frac{\text{Apparent height}}{\text{Real height}}$$

$$\therefore AI'' = \text{Apparent height} = \mu \times \text{real height} = \frac{4}{3} \times 75 = 100 \text{ cm.}$$

24. 2

Sol. $\frac{q_1}{C_1} = \frac{q_2}{C_2}; q_1 + q_2 = 2Q_0$

$$C_1 = \frac{\epsilon_0 A}{d_0 + vt}; C_2 = \frac{\epsilon_0 A}{d_0 - vt}$$



$$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$$

$$q_2 \left(\frac{d_0 - vt}{d_0 + vt} \right) + q_2 = 2Q_0$$

$$q_2 \left[\frac{2d_0}{d_0 + vt} \right] = 2Q_0$$

$$q_2 = \frac{2Q_0}{2d_0} (d_0 + vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 2 \text{ amp}$$

25. 750

Sol. $e = \frac{v_2 - v_1}{u_1 - u_2}$

$$1 = \frac{v_2 - (-2)}{u_1}$$

$$u_1 = v_2 + 2$$

$$u_1 = 1(-2) + 5(u_1 - 2)$$

$$u_1 = -2 + 5u_1 - 10$$

$$u_1 = \frac{12}{4} = 3 \text{ m/s}$$

$$v_2 = 1 \text{ m/s}$$

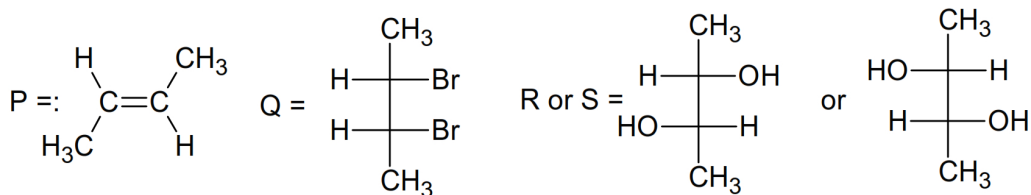
$$\text{Kinetic energy of the centre of mass} = \frac{1}{2} \times (1+5) \times \left(\frac{3}{1+5} \right)^2 = \frac{3}{4} \text{ J.}$$

Chemistry

PART – B

SECTION – A

26. D
Sol.



27. C

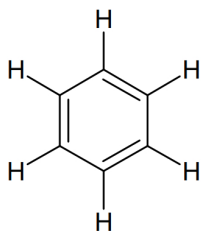
Sol. $2\pi r = n\lambda \Rightarrow \lambda = \frac{2\pi r}{n} = \frac{2\pi n^2 r_0}{n \times z} = 2\pi n r_0 = 2\pi \times 4 \times r_0 = 8\pi r_0$

28. B

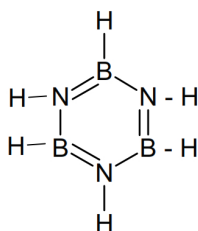
Sol. $\text{CO}_3^{2-} + \text{H}^+ \longrightarrow \text{HCO}_3^-$ (Weak base does not dissociate much). Therefore, the reaction proceeds to forward direction by removing CO_3^{2-} as HCO_3^- .

29. D

Sol. Organic benzene



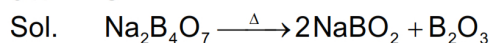
Inorganic benzene



30. D

Sol. The nearer the -I groups towards COOH group, stronger is the acid.

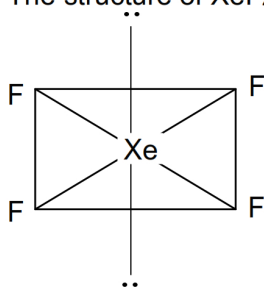
31. C



32. B

Sol. A, C, D will show geometrical isomerism. (B) will show geometrical as well as optical isomerism.

33. C
Sol. The structure of XeF_4 is:



34. C
Sol. For zero order reaction,
Rate = k
Both rate and rate constant are independent of concentration.

35. D
Sol. $\text{pH of NaH}_2\text{PO}_4 = \frac{\text{p}K_{a1} + \text{p}K_{a2}}{2} = \frac{7.2 + 10.6}{2} = 8.9$

36. C
Sol. In NH_4Cl , the oxidation state of nitrogen is -3.

37. B
Sol. In case of weak acid heat of neutralization is less than $-13.7 \text{ KCal / mole}$ because extra energy is required to dissociate the acid.

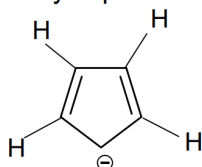
38. D
Sol. Sulphur forms two sigma bonds with two lone pairs.

39. B
Sol. $\frac{\Lambda_m}{\Lambda_e} = 3 \Rightarrow \Lambda_m = 3\Lambda_e$
n-factor of $\text{MgCl}_2 \cdot \text{KCl} \cdot 6 \text{ H}_2\text{O} = 3$.

40. A
Sol. Aliphatic amines are stronger base than aromatic amines

41. B
Sol. In (B) the NH_2 group which is at a larger distance from the COOH group, should be more basic it should accept H^+ ion from acid.

42. C
Sol. In cyclopentadienyl anion, each carbon is sp^2 hybridized



43. B
Sol. Halogen atoms except fluorine form compounds with valency 1, 3, 5 and 7.

44. C

Sol. The fraction is $\frac{2}{3}$ because out of three chlorine atoms only two are precipitated.

45. C

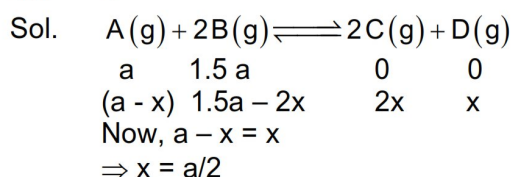
Sol. Increases in pressure or addition of C affect the second equilibrium hence concentration of A_2 changes which further affect the first equilibrium. Addition of inert gas at constant volume does not affect the equilibrium.

SECTION – B

46. 4

Sol.
$$\text{pH} = \frac{1}{2}(\text{pK}_a - \log C)$$
$$= \frac{1}{2}(5 - \log 10^{-3}) = \frac{1}{2}(5 + 3) = 4$$

47. 4

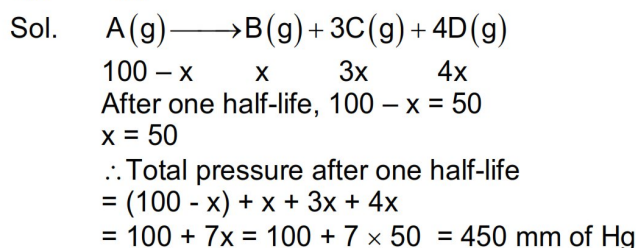


$$\therefore = K_c \frac{[\text{C}]^2 (\text{D})}{[\text{A}][\text{B}]^2} = \frac{\left(\frac{a}{2}\right)(a)^2}{\left(\frac{a}{2}\right)(0.5a)^2} = 4$$

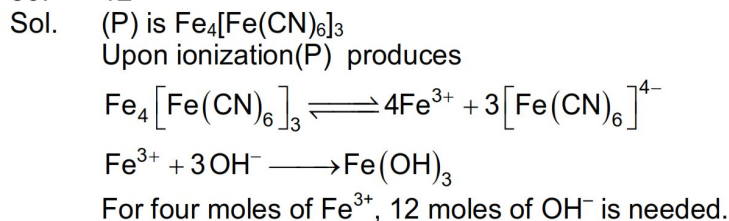
48. 50

Sol. The possible orbitals for the 9th period are 9s, 9p, 8d, 7f, 6g. The number of electrons that can be accommodated in the orbitals are $2 + 6 + 10 + 14 + 18 = 50$

49. 450



50. 12



SECTION – A

51. D

Sol. Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100} \right] n$ where $[n]$ is greatest integer function $= \left[0.33 + \frac{3n}{100} \right] n$

For $n = 1, 2, \dots, 22$, we get $f(n) = 0$ and for $n = 23, 24, \dots, 55$, we get $f(n) = 1 \times n$

For $n = 56$, $f(n) = 2 \times n$

$$\text{So } \sum_{n=1}^{56} f(n) = 1(23) + 1(24) + \dots + 1(55) + 2(56)$$

$$= (23 + 24 + \dots + 55) + 112 = \frac{33}{2} [46 + 32] + 112$$

$$= \frac{33}{2} (78) + 112 = 1399$$

52. A

Sol. Given equation is, $x^2 + x \sin \theta - 2 \sin \theta = 0$

$$\alpha + \beta = -\sin \theta \text{ and } \alpha\beta = -2 \sin \theta$$

$$\frac{(\alpha^{12} + \beta^{12}) \alpha^{12} \beta^{12}}{(\alpha^{12} + \beta^{12})(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$$

$$\therefore |\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{\sin^2 \theta + 8 \sin \theta}$$

$$\therefore \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}} = \frac{(2 \sin \theta)^{12}}{\sin^{12} \theta (\sin \theta + 8)^{12}} = \frac{2^{12}}{(\sin \theta + 8)^{12}}$$

53. B

Sol. Let $I = \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$

$$I = \int \left(\frac{\sin x \cdot \cos x}{\sin^3 x + \cos^3 x} \right)^2 dx$$

$$I = \int \left(\frac{\sin x \cdot \cos x}{\cos^3 x (1 + \tan^3 x)} \right)^2 dx = \int \left(\frac{\sin x \cdot \sec^2 x}{(1 + \tan^3 x)} \right)^2 dx$$

$$\text{Put } 1 + \tan^3 x = t$$

$$dt = -3 \tan^2 x \sec^2 x dx \text{ or } dx = \frac{dt}{3 \tan^2 x \sec^2 x}$$

$$\therefore I = \int \frac{\sin^2 x \cdot \sec^4 x}{t^2} \times \frac{dt}{3 \tan^2 x \sec^2 x}$$

$$\begin{aligned}
 I &= \frac{1}{3} \int \frac{\sin^2 x \cdot \sec^4 x}{t^2} \times \frac{dt}{\frac{\sin^2 x}{\cos^2 x} \times \sec^2 x} \\
 &= \frac{1}{3} \int \frac{\sin^2 x \cdot \sec^4 x}{t^2} \times \frac{dt}{\sin^2 x \sec^4 x} \\
 \therefore I &= \frac{1}{3} \int \frac{dt}{t^2} = \frac{1}{3} \int t^{-2} dt \\
 \Rightarrow I &= \frac{1}{3} \left[\frac{t^{-2+1}}{-2+1} \right] + c = \frac{-1}{3} \left[\frac{1}{t} \right] + c \\
 \text{or } I &= -\frac{1}{3(1+\tan^3 x)} + c
 \end{aligned}$$

54. D

Sol. $P(4t^2 + 3, 8t^3 - 1)$

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{dy}{dt} \cdot \frac{dt}{dx} = 3t \quad (\text{slope of tangent at } P) \\
 \frac{dt}{dx} &= \frac{dy}{dx} = 3t
 \end{aligned}$$

Let $Q = (4\lambda^2 + 3, 8\lambda^3 - 1)$

slope of PQ = 3t

$$\frac{8t^3 - 8\lambda^3}{4t^2 - 4\lambda^2} = 3t \Rightarrow t^3 - 3\lambda^2 t + 2\lambda^3 = 0$$

$$(t - \lambda) \cdot (t^2 + t\lambda - 2\lambda^2) = 0$$

$$(t - \lambda)^2 \cdot (t + 2\lambda) = 0 \Rightarrow t = \lambda \quad (\text{or}) \quad \lambda = \frac{-t}{2}$$

$$\therefore Q[t^2 + 3, -t^3 - 1]$$

55. D

Sol. For infinitely many solution

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 1(10) - 1(7) + k(-1) - 0 \Rightarrow k = 3$$

$$k = 3, \text{ 2nd system is } 4x + 5y = 7$$

$$\text{and } 7x + 8y = 10$$

Clearly, they have a unique solution

$$(ii) - (i) \Rightarrow 3x + 3y = 3 \Rightarrow x + y = 1$$

.....(i)

.....(ii)

56. C

Sol. Give $\cos^{-1}(2x) - 2\cos^{-1}\sqrt{1-x^2} = \pi$

$$\cos^{-1}(2x) - \cos^{-1}(2(1-x^2)-1) = \pi$$

$$\cos^{-1}(2x) - \cos^{-1}(1 - 2x^2) = \pi$$

$$-\cos^{-1}(1 - 2x^2) = \pi - \cos^{-1}(2x)$$

Taking cos both sides we get

$$\cos(-\cos^{-1}(1 - 2x^2)) = \cos(\pi - \cos^{-1}(2x))$$

$$1 - 2x^2 = -2x \Rightarrow 2x^2 - 2x - 1 = 0$$

$$\text{On solving, } x = \frac{1 \pm \sqrt{3}}{2}$$

$$\text{As } x = \left[\frac{-1}{2}, \frac{1}{2} \right], x = \frac{1 + \sqrt{3}}{2} = \text{rejected}$$

$$\text{So } x = \frac{1 - \sqrt{3}}{2} \Rightarrow x^2 - 1 = \frac{-\sqrt{3}}{2}$$

$$= 2 \sin^{-1}(x^2 - 1) = 2 \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-2\pi}{3}$$

57. B

$$\text{Sol. Since, } a_1 + a_3 = 10 = a_1 + d \Rightarrow 5 \text{ and } \frac{1}{6} \sum_{i=1}^6 a_i = \frac{19}{2}$$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$$

$$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57 \Rightarrow a_1 + a_6 = 19$$

$$\Rightarrow 2a_1 + 5d = 19 \text{ and } a_1 + d = 5 \Rightarrow a_1 = 2, d = 3$$

Numbers : 2, 5, 8, 11, 14, 17

Then, variance = σ^2 = mean of squares – square of mean

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}; \text{So, } 8\sigma = 210$$

58. C

$$\text{Sol. Given } \beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$$

$$\beta = \lim_{x \rightarrow 0} \frac{1 + \alpha x - \left[1 + 3x + \frac{9x^2}{2!} + \dots\right]}{(\alpha x) \frac{(e^{3x} - 1)}{3x} 3x}$$

$$\beta = \lim_{x \rightarrow 0} \frac{(\alpha x - 3x) - \frac{9x^2}{2!} - \dots}{3\alpha x^2}$$

For existence of limit $\alpha - 3 = 0, \alpha = 3$

$$\text{Limit } \beta = \frac{-3}{2\alpha}, \beta = -\frac{1}{2}$$

$$\text{Now, } \alpha + \beta = \frac{5}{2}$$

59. D

Sol. Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$

$$\text{Then, } e_1 = \sqrt{1 - \frac{b^2}{25}}$$

The equation of hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$

$$\text{Then, } e_2 = \sqrt{1 + \frac{b^2}{16}}, e_1 e_2 = 1$$

$$\Rightarrow (e_1 e_2)^2 = 1 \Rightarrow \left(1 - \frac{b^2}{25}\right) \left(1 + \frac{b^2}{16}\right) = 1$$

$$\Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25 \times 16} = 1$$

$$\Rightarrow \frac{9}{16 \cdot 25} b^2 - \frac{b^4}{25 \cdot 16} = 0 \Rightarrow b^2 = 9 \quad \therefore e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\text{And, } e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$$

Distance between foci of ellipse $= \alpha = 2ae_1 = 2(5)(e_1) = 8$

Distance between foci of hyperbola

$$= \beta = 2ae_2 = 2(4)(e_2) = 10 \therefore (\alpha, \beta) = (8, 10)$$

60. A

Sol. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a > b$, $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent $\equiv y = mx \pm \sqrt{a^2 m^2 + b^2}$

$$\text{Comparing with } y = \frac{-x}{6} + \frac{4}{3}$$

$$m = \frac{-1}{6} \text{ and } a^2 m^2 + b^2 = \frac{16}{9}$$

$$\Rightarrow \frac{a^2}{36} + \frac{4}{9} = \frac{16}{9} \Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{9} = \frac{4}{9}$$

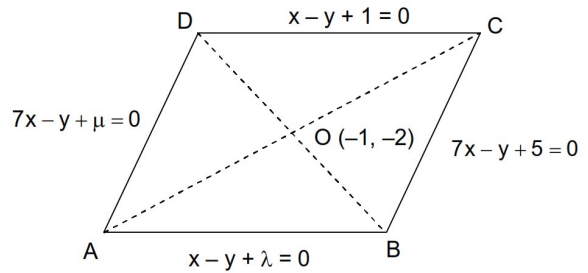
$$\Rightarrow a^2 = 16 \Rightarrow a = \pm 4$$

Now, eccentricity of ellipse $(e) = \sqrt{1 - \frac{b^2}{a^2}}$

$$\Rightarrow e = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}} = \frac{1}{2} \sqrt{\frac{11}{3}}$$

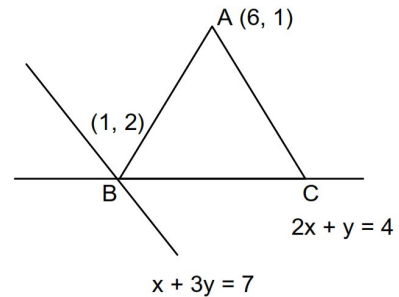
61.
Sol.

A
Let other two sides of rhombus are $x - y + \lambda = 0$ and $7x + y + \mu = 0$ then O is equidistant from AB and DC and from AD and BC
 $\therefore |-1 + 2 + 1| = |-1 + 2 + \lambda| \Rightarrow \lambda = -3$ and
 $|-7 + 2 - 5| = |-7 + 2 + \mu| \Rightarrow \mu = 15$
 $\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$
 $\therefore$ On solving the equations of sides pair wise, we get the vertices as
 $\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$



62.
Sol.

C
Intersection of two lines in point B with coordinate (1, 2). As C lies on $2x + y = 4$, suppose $x = h$ then $k = 4 - 2h$.
 Δ is isosceles with base BC. Then, $AB = AC$
 $\sqrt{25 + 1} = \sqrt{(6 - h)^2 + (3 - 2h)^2}$
 $\sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$
Take square both sides
 $26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$
 $\Rightarrow 5h^2 - 5h - 19h + 19 = 0$
 $\Rightarrow 5h^2 - 24h + 19 = 0$
 $h = \frac{19}{5}$ or $h = 1$.



Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid (α, β) of triangle

$$ABC = \left(\frac{6 + 1 + \frac{19}{5}}{3}, \frac{1 + 2 - \frac{18}{5}}{3} \right)$$

$$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$$

Put the value of α and β in $15(\alpha + \beta)$.

$$15(\alpha + \beta) = 51$$

63.
Sol.

C
By splitting given equation

$$\frac{1}{20} \left[\left(\frac{1}{20 - a} - \frac{1}{40 - a} \right) + \left(\frac{1}{40 - a} - \frac{1}{60 - a} \right) + \dots + \left(\frac{1}{180 - a} - \frac{1}{200 - a} \right) \right] = \frac{1}{256}$$

$$\Rightarrow \frac{1}{20} \left(\frac{1}{20-a} - \frac{1}{200-a} \right) = \frac{1}{256}$$

$$\Rightarrow (20-a)(200-a) = 256 \times 9$$

$$\Rightarrow a^2 - 220a + 1696 = 0 \Rightarrow a = 8, 212$$

$$\Rightarrow \text{Hence, maximum value of } a \text{ is } 212$$

64. C

Sol. $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$

$$= (1+x)^{10} \frac{\left[1 - \left(\frac{x}{1+x} \right)^{11} \right]}{\left(1 - \frac{x}{1+x} \right)}$$

$$\Rightarrow (1+x)^{11} - x^{11}$$

Coefficient of x^7 is $^{11}C_7 = 330$

65. B

Sol. Let $3^x = y$

$$\therefore y(y-1) + 2 = |y-1| + |y-2|$$

Case I : When $y > 2$

$$y^2 - y + 2 = y - 1 + y - 2 \Rightarrow y^2 - 3y + 5 = 0$$

$\therefore D < 0$ [$\therefore$ Equation not satisfy]

Case II : When $1 \leq y \leq 2$

$$y^2 - y + 2 = y - 1 - y + 2; y^2 - y + 1 = 0$$

$\therefore D < 0$ [$\therefore$ Equation not satisfy]

Case III : When $y \leq 1$

$$y^2 - y + 2 = -y + 1 - y + 2 \Rightarrow y^2 + y - 1 = 0$$

$$\therefore y = \frac{-1 + \sqrt{5}}{2} = \frac{-1 - \sqrt{5}}{2} \quad [\therefore \text{Equation not satisfy}]$$

$$\therefore \text{only one } -1 + \frac{\sqrt{5}}{2} \text{ satisfy equation}$$

66. C

Sol. $\therefore \sum_{k=0}^{10} (3k+4) {}^{10}C_k = \sum_{k=1}^{10} 3k \cdot \frac{10}{k} {}^9C_{k-1} + 4 \sum_{k=0}^{10} {}^{10}C_k$

$$= 30 \sum_{k=1}^{10} {}^9C_{k-1} + 4 \sum_{k=0}^{10} {}^{10}C_k$$

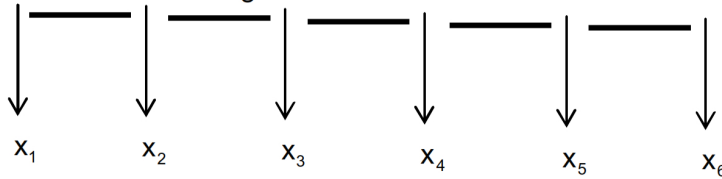
$$= 30 \cdot 2^9 + 4 \cdot 2^{10} \left[\therefore \sum_{r=0}^n {}^nC_r = 2^n \right]$$

$$= 19 \cdot 2^{10} \Rightarrow \alpha = 0 \text{ and } \beta = 19.$$

So, $\alpha + \beta = 19$

67.
Sol.

C
Total 16 cubes are given and out of which 11 are blue and 5 are red.



$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$

$$x_1, x_6 \geq 0, x_2, x_3, x_4, x_5 \geq 2$$

$$x_2 = t_1 + 2 \Rightarrow x_3 = t_3 + 2 \Rightarrow x_4 = t_4 + 2 \Rightarrow x_5 = t_5 + 2$$

$$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$$

$$\text{Number of solutions} = {}^{6+3-1}C_3 = {}^8C_3 = 56$$

68. A

Sol. Given, $|\vec{a} + \vec{b}| = \sqrt{3} \Rightarrow \cos \theta = \frac{1}{2}$ or $\theta = \frac{\pi}{3}$

$$\text{We have } \vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$$

$$|\vec{c}| = \sqrt{(\vec{a})^2 + (2\vec{b})^2 + \left(\frac{3\sqrt{3}}{2}\hat{n}\right)^2 + 4\vec{a} \cdot \vec{b} + 6\sqrt{3}\hat{n} \cdot \vec{b} + 3\sqrt{3}\hat{n} \cdot \vec{a}}$$

$$= \sqrt{1 + 4 + \frac{27}{4}|\hat{n}|^2 + 4 \cdot \frac{1}{2} + 0 + 0} \quad [\because \hat{n} \perp \vec{b} \text{ and } \hat{n} \perp \vec{a}]$$

$$(\vec{c}) = \frac{\sqrt{55}}{2}$$

69. B

Sol. Given $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$

Multiply $\vec{b}$ both sides,

$$\vec{b} \cdot \vec{c} = \vec{b} \cdot (2\vec{a} \times \vec{b}) - 3|\vec{b}|^2$$

$$\text{Since, } \vec{b} \cdot (2\vec{a} \times \vec{b}) = 0 \text{ then } \vec{b} \cdot \vec{c} = -3|\vec{b}|^2$$

$$\vec{b} \cdot \vec{c} = -3 \times 16 = -48$$

70. B

Sol. $\vec{a} \cdot \vec{b} = 1 \Rightarrow (\alpha\hat{i} + \beta\hat{j} + 3\hat{k})(-\beta\hat{i} - \alpha\hat{j} - \hat{k}) = 1$

$$\Rightarrow -\alpha\beta - \alpha\beta - 3 = 1 \Rightarrow \alpha\beta = -2 \quad \dots\dots(i)$$

$$\vec{b} \cdot \vec{c} = -3 \Rightarrow (-\beta\hat{i} - \alpha\hat{j} - \hat{k})(\hat{i} - 2\hat{j} - \hat{k}) = -3$$

$$\Rightarrow -\beta + 2\alpha + 1 = -3 \Rightarrow 2\alpha - \beta = -4 \quad \dots\dots(ii)$$

From (i) and (ii), $\alpha = -1, \beta = 2,$

$$\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c}) = \frac{1}{3} \begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = 2$$

SECTION - B

71. 4

Sol. $x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$

Given equation is

$$14 \operatorname{cosec}^2 x - 2 \sin^2 x = 21 - 4 \cos^2 x$$

$$= 21 - 4(1 - \sin^2 x) = 17 + 4 \sin^2 x$$

$$14 \operatorname{cosec}^2 x - 6 \sin^2 x = 17$$

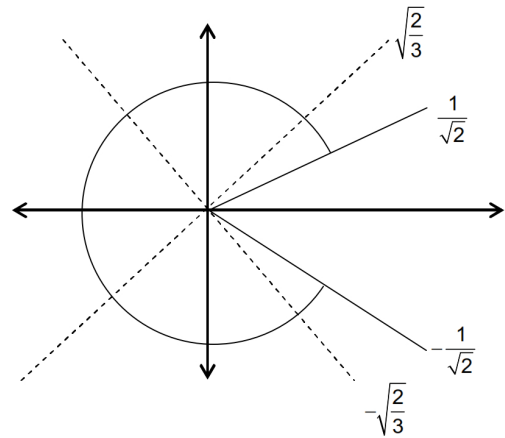
Let $\sin^2 x = t$

$$\frac{14}{t} - 6t = 17 \Rightarrow 14 - 6t^2 = 17t$$

$$6t^2 + 17t - 14 = 0 \Rightarrow t = -3, 5, \frac{2}{3} \Rightarrow \sin^2 x = \frac{2}{3}$$

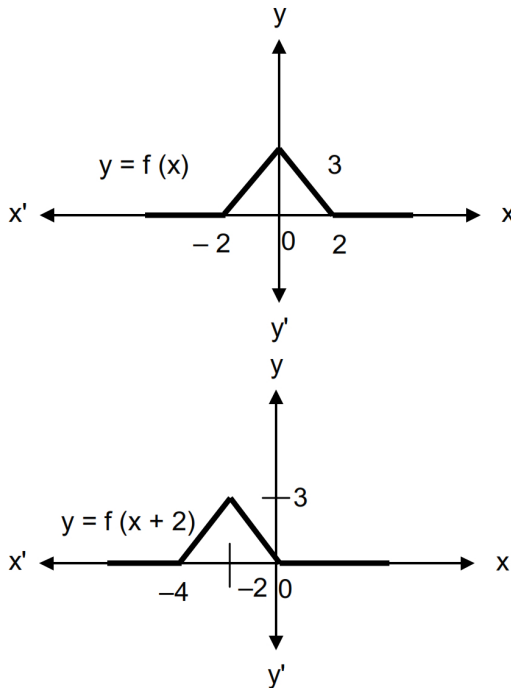
$$\Rightarrow \sin x = \pm \sqrt{\frac{2}{3}}$$

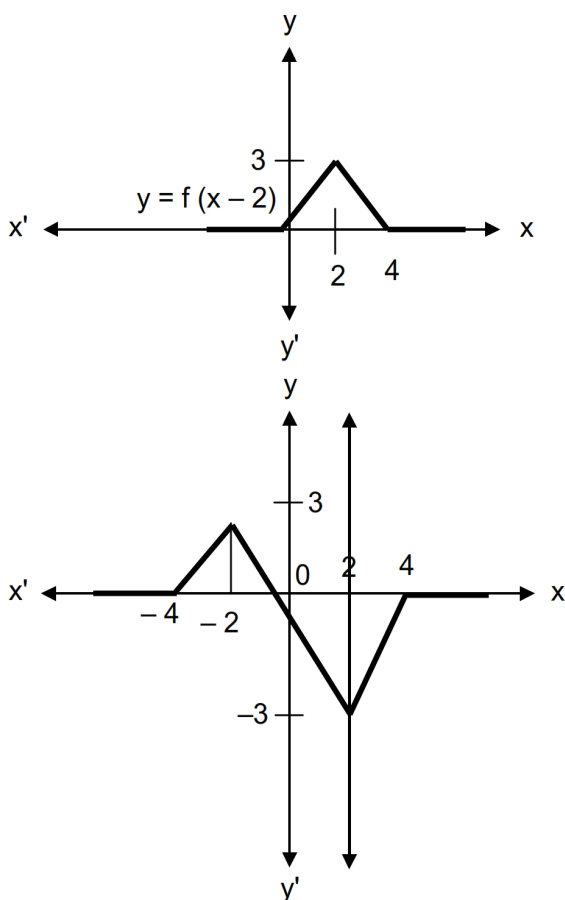
$\therefore$ Total 4 solutions



72. 4

Sol.
$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } -2 \leq x \leq 2 \\ 0 & \text{if } x \in (-\infty, -2) \cup (2, \infty) \end{cases}$$





73. 4

Sol. $\therefore A^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix}, A^3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^3 & 0 \\ 3 & 0 & -1 \end{vmatrix}, A^4 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^4 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$\text{So, } A^{20} + \alpha A^{19} + \beta A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{vmatrix} + \alpha \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{vmatrix} + \beta \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20} + \alpha 2^{19} + 2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

On comparing, we get

$$\alpha + \beta = 0 \text{ and } 2^{20} + \alpha \cdot 2^{19} + 2\beta = 4 \Rightarrow \alpha = -2 \text{ and } \beta = 2$$

$$\text{So, } \beta - \alpha = 2 + 2 = 4$$

74. 19

Sol. $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ $3, 7, 9 \rightarrow \text{odd}$, $0, 4, 6, 8, 10 \rightarrow$
even

$$R = \{x - y = \text{odd} + \text{ve or } x - y = 2\}$$

$R = \{(6, 4), (8, 6), (9, 7), (10, 8), (3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 4), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)\}$
 minimum ordered pairs to be added must be $15 + 4 = 19$

75. 2

Sol. $x^2 + y^2 - 4x - 2y + 5 - \alpha = 0$

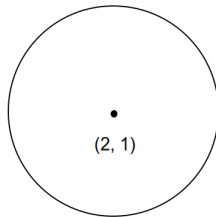
$C_1(2, 1)$ and $r_1 = \sqrt{\alpha}$

$2x - y + 1 = 0$

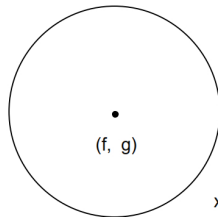
Image of $(2, 1)$ in given line will be

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$



$$L = 2x - y + 1 = 0$$



$$x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$$

$$\Rightarrow x = 2 - \frac{16}{5} = \frac{-6}{5}, y = 1 + \frac{8}{5} = \frac{13}{5}$$

$$\text{So, } x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0, C_2(f, g) \text{ and } r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$$

[radius of both circle will be same]

$$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5}$$

$$\left[\because f = -\frac{6}{5}, g = \frac{13}{5} \right]$$

$$= \frac{36 + 169 - 180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$$

$$\therefore \alpha + r = 2$$